Estimating Sentiment, Risk Aversion, and Time Preference from Behavioral Pricing Kernel Theory

Abstract

We apply behavioral pricing kernel theory to estimate aggregate preferences, beliefs, and sentiment from option prices and historical returns. In contrast to non-behavioral empirical pricing kernel analysis, our approach to estimating risk aversion and time preference explicitly controls for sentiment. We find that sentiment, risk aversion, and time preference exhibit strong behavioral patterns across the business cycle, with significant implications for the co-movement of risk and return.For the purpose of corroborating our findings, we compare our estimates to a series of independent variables discussed in the literature.

Behavioral pricing kernel theory

Behavioral Pricing Kernel Theory（行為定價核理論）是行為金融學（Behavioral Finance）對於傳統定價核理論（Pricing Kernel Theory）的擴展，它引入了投資者的**行為偏誤（Behavioral Biases）**，解釋市場價格與風險補償的異常現象。

在傳統的資產定價理論中，**定價核（Pricing Kernel, SDF）**由投資者的邊際效用決定，其斜率衡量風險趨避程度。但行為金融學認為，**投資者並非完全理性，他們的行為偏誤會影響定價核的形狀，進而影響資產價格**。

- **行為金融學**發現，市場的實際定價核**並不總是單調遞減**，在某些市場環境下可能出現 **倒 U 形（Inverted U-Shape）**，這與行為偏誤有關。
- 行為定價核（Behavioral Pricing Kernel）考慮了投資者的心理因素，例如：
    1. **過度自信（Overconfidence）**
    2. **損失厭惡（Loss Aversion）**
    3. **尾部情緒（Tail Sentiment）**
    4. **機率權重（Probability Weighting）**

這些行為偏誤導致**市場錯誤定價（Mispricing）**，並影響資產的風險溢價（Risk Premium）。

Behavioral pricing kernel theory

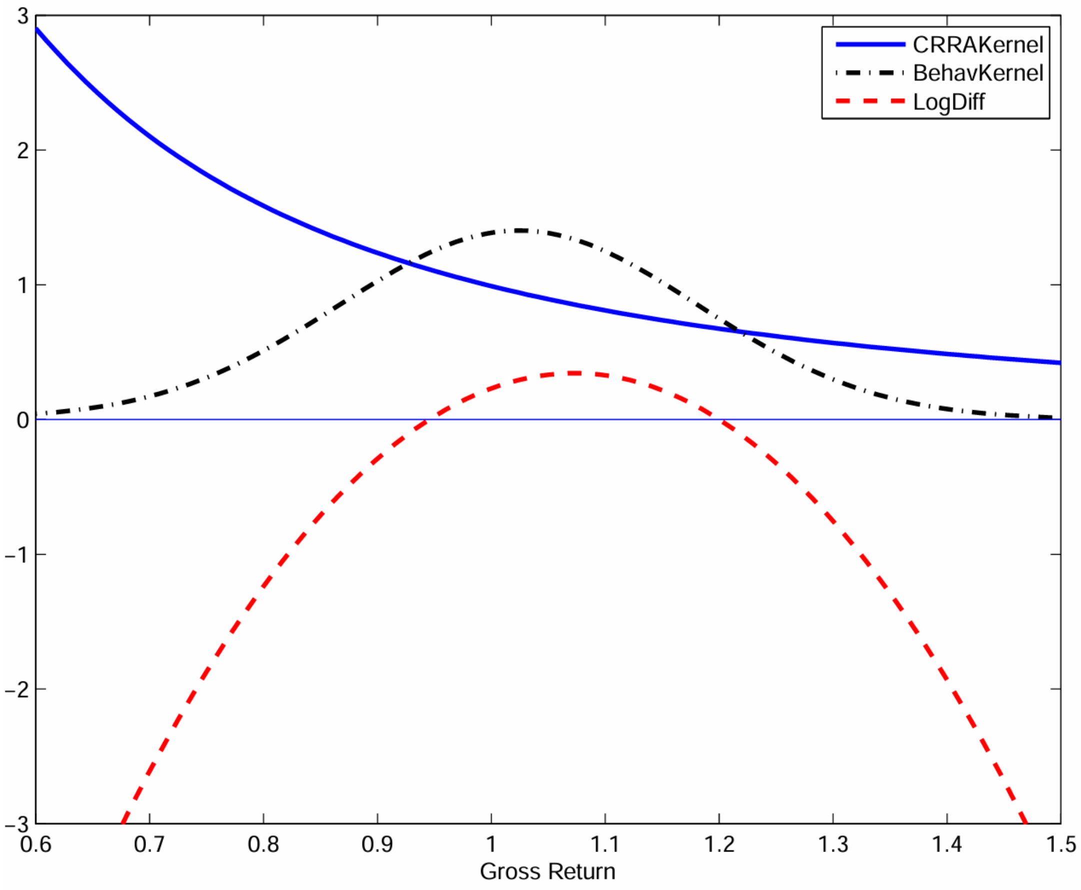
The function CRRAKernel

- The pricing kernel from a neoclassical representative investor model with CRRA preferences.
- As usual, the function is monotonically decreasing
- Reflects intertemporal marginal rate of substitution through function slope
- Measures time preference (through function height)

The function BehavKernel

A pricing kernel associated with a representative investor whose beliefs exhibit excessive optimism and overconfidence

To compute the differences between the two pricing kernels, we use the log of BehavKernel minus the log of CRRAKernel, which is displayed as the function LogDiff



Estimating steps

- First, we apply the FHS procedure to index option data to estimate a weekly series of empirical pricing kernels.
- Second, we decompose each pricing kernel into a fundamental component and a sentiment component.
- Third. we use the implied change of measure in the sentiment components, together with GARCH estimates of conditional return distributions to obtain weekly estimates of specific biases such as excessive optimism, overconfidence, and tail sentiment.
- Fourth, we compare our estimates of these biases to variables in the literature.
- Fifth, we compare our estimates of market risk aversion and time preference to estimates of these variables that have been obtained by other means.

Estimating steps—estimating series of empirical pricing kernels

$$M_{t,T} = e^{-r\,(T-t)} \frac{q(S_T/S_t)}{p(S_T/S_t)}$$

q is the risk neutral density, **p the objective or historical density**, r the instantaneous risk free rate, and St the S&P 500 index at date t, which is a proxy for the market portfolio.

Using the empirical method in Barone-Adesi, Engle, and Mancini (2008) (BEM)
→two GARCH models are estimated, the two GARCH models are effectively used as “filters” to extract the **conditional risk neutral and objective densities of market returns**, respectively.
→Once the conditional densities q and p are estimated, the SDF is recovered by taking their discounted ratio
→ We are interested in a long and fixed time horizon of one year, and there are virtually no options with time to maturity of one year for each day t. This motivates our approach of using the BEM method and estimating risk neutral GARCH models from the cross-section of options.

將BEM方法替換成CDI method
→ CDI 方法適用於估計短時間pricing kernel
→分析短時間內投資人情緒變化

Estimating steps—decomposing pricing kernel

decomposing each pricing kernel into a fundamental component and a sentiment component.

Shefrin (2005, 2008), Bhamra and Uppal (2014, 2015), and others, theoretically show that investors' sentiment induce deviations between **empirical** and **neoclassical pricing kernels**.

Neoclassical pricing kernels
→we start with a **CRRA SDF**, which is the standard benchmark in neoclassical theory.

CRRA SDF

$$M_{t,T}(\theta) = \theta_0 \left(\frac{S_T}{S_t}\right)^{-\theta_1}$$

θ_0 is a discount factor measuring the degree of impatience, θ_1 is the coefficient of relative risk aversion, and $\theta = (\theta_0 , \theta_1)$

$$\rightarrow \log\left(M_{t,T}(\theta)\right) = \log(\theta_0) - \theta_1 \log\left(\frac{S_T}{S_t}\right) \; (*)$$

empirical SDF

$M_{t,T}$

→Using log-log regression model $\log(M_{t,T}) = \beta_0 + \beta_1 \log\left(\frac{S_T}{S_t}\right)$ to get estimate of $\log(\theta_0)$ and θ_1

→ Using $\widehat{\log(\theta_0)}$ and $\widehat{\theta_1}$, allowing us to estimate $\log\left(M_{t,T}(\theta)\right)$

→ $d_{t,T} = \log(M_{t,T}) - \log\left(M_{t,T}(\theta)\right)$ = estimate of sentiment function ($\Lambda_{t,T}$)

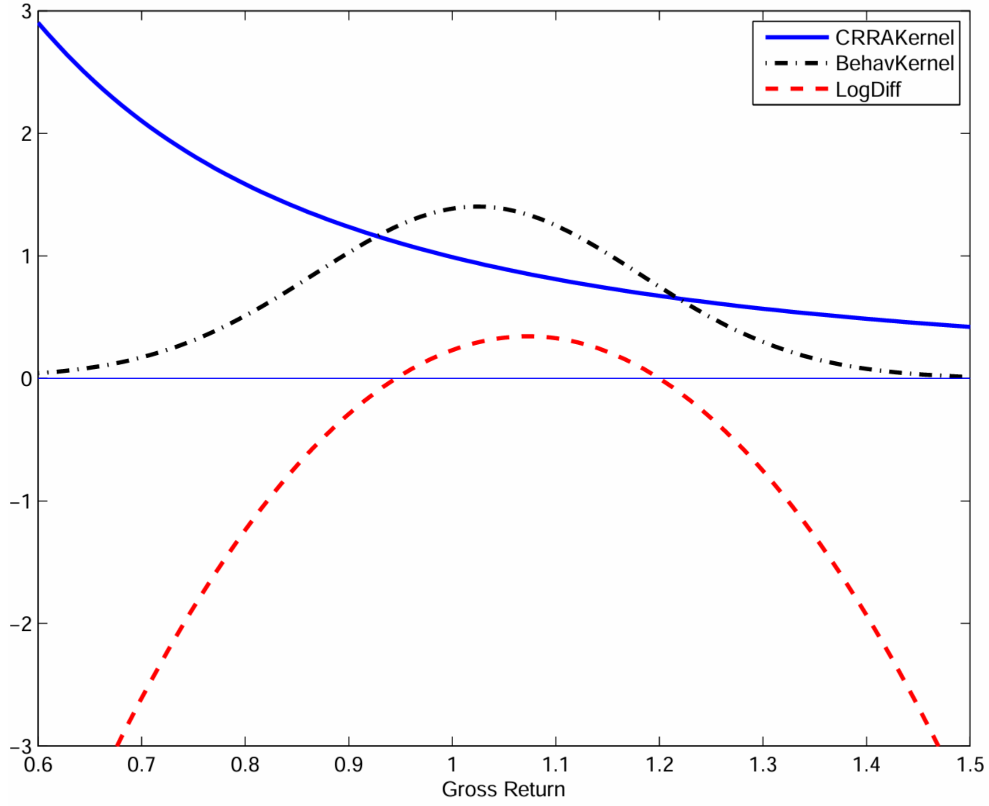
In Shefrin (2008), (*) generalizes to include an additional term $\Lambda_{t,T}$ to reflect the impact of sentiment.

$$\log(M_{t,T}) = \Lambda_{t,T} + \log(\theta_0) - \theta_1 \log\left(\frac{S_T}{S_t}\right) \rightarrow M_{t,T} = e^{\Lambda_{t,T}} \, \theta_0 \left(\frac{S_T}{S_t}\right)^{-\theta_1}$$

Estimating steps—decomposing pricing kernel

decomposing each pricing kernel into a fundamental component and a sentiment component.

1. For each day t, we obtain a grid of 100 values of gross returns, $\frac{S_T^i}{S_t}$, i = 1, . . . , 100, spanning the support of the empirical SDF.
2. Regress the empirical log-SDF, $\log(M_{t,T}^i)$, on a constant and the log gross return, $\log(\frac{S_T^i}{S_t})$. Intercept and slope provide estimates of $\log(\theta_{0,t})$ and $-\theta_{i,t}$, respectively, allowing us to estimate $\log\left(M_{t,T}(\theta)\right)$
3. For each gross return $\frac{S_T^i}{S_t}$, we compute the pointwise difference $d_{t,T}^i$, provide an estimate of the sentiment function $\Lambda_{t,T}$ over the support of gross returns, $\frac{S_T^i}{S_t}$, i = 1, . . . , 100.
4. Repeat this procedure for each day t, and obtain a time series of $\theta_{0,t}$, $\theta_{i,t}$ and the sentiment functions $\Lambda_{t,T}$.



Estimating steps—sentiment function

實證定價核應是$\log(\mathrm{M}_{\mathrm{t,T}}) = \log(\theta_0) - \theta_1 \log\left(\frac{\mathrm{S_T}}{\mathrm{S_t}}\right) + \Lambda_{t,T}$， $\log(\theta_0) - \theta_1 \log\left(\frac{\mathrm{S_T}}{\mathrm{S_t}}\right)$表示基本面反映的理論定價核估計項， $\Lambda_{t,T}$為投資人情緒估計項。

$\log(\mathrm{M}_{\mathrm{t,T}})$與$\log\left(\frac{\mathrm{S_T}}{\mathrm{S_t}}\right)$ log-log regression 估計θ_0、 θ_1，找出$\log\left(\mathrm{M}_{\mathrm{t,T}}(\theta)\right) = \log(\theta_0) - \theta_1 \log\left(\frac{\mathrm{S_T}}{\mathrm{S_t}}\right)$

$\log(\mathrm{M}_{\mathrm{t,T}})$- $\log\left(\mathrm{M}_{\mathrm{t,T}}(\theta)\right)$可估計$\Lambda_{t,T}$，運用$\Lambda_{t,T}$可將客觀機率密度函數$P$轉換為主觀機率密度函數$P_R$

文獻中以歷史S&P 500資料估計客觀機率密度函數P，計算定價核提取情緒估計項後轉換為主觀機率密度函數P_R

$\Lambda_{t,T}$

This function is a scaled log-change of measure, where the change of measure transforms the objective pdf P into the representative investor's pdf P_R

市場情緒會影響投資者對未來事件機率的主觀認知，使其偏離客觀機率。

$e^{\Lambda_{t,T}}$is proportional to the change of measure $\frac{P_R}{P}$: $P_R = Pe^{\Lambda_{t,T}} \frac{\theta_{0,t,P}}{\theta_{0,,t}}$, where $\theta_{0,t,P}$ is a rescaling of $\theta_{0,t}$ whose purpose is to ensure that P_R integrates to one. $\frac{\theta_{0,t,P}}{\theta_{0,,t}}$ 表示需要標準化來確保機率密度性質

Representative Investor's Beliefs, Optimism and Overconfidence

Optimism is defined as the difference between the expected market return under the representative investor's and objective pdfs,

$$E_t^{p_R}[S_T/S_t] - E_t^{p}[S_T/S_t]$$

where E_t^p is the conditional expectation under the objective pdf at date t, computed by numerically integrating the gross return

$\frac{S_T}{S_t}$ against p, and similarly for $\mathrm{E}_t^{p_R}$.

Overconfidence is defined as the difference between the expected volatility of the market return under objective and representative investor's pdfs,

$$\sqrt{\mathrm{Var}_t^{p}[S_T/S_t]} - \sqrt{\mathrm{Var}_t^{p_R}[S_T/S_t]}.$$